

The Borromean Knot

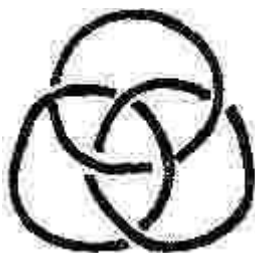
Chapter7: The Borromean Knot

The Borromean knot is a certain way of knotting loops of thread.

Mathematicians have only very recently begun to occupy themselves with the part of human experience constituted by the art of knots. This discordance is astonishing. On the carpenter's chain at the origin of Egyptian mathematics, knots were already utilized to mark measurements. Every year, after the rising of the Nile, it was necessary to re-measure the fields, and they relied on knots to do so. This is to say that the knot was part of a very ancient, practical, realistic, concrete human experience, of which mathematics is a theoretical and logical reflection. It was not until the 20th century that mathematicians began to take an interest in knots. They then defined different forms of knots and chains, according to their qualities of knotting and unknotting. The Borromean knot retains a particular place in these studies.

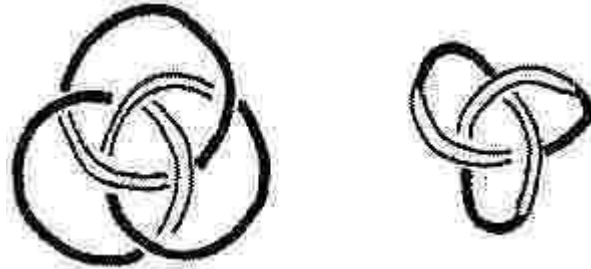
It must be remarked that is an abuse of language to speak of the Borromean knot to designate this chain of three threads:

A knot is in fact formed by single thread that follows a particular enough trajectory not to be reducible to a simple round. On the other hand, as soon as there are several threads in play, one speaks of a chain.



We speak of a Borromean knot to designate a Borromean chain.

Among all the chains existent or imaginable, the Borromean knot holds a place apart, because the threads constituting it are held together by a knot, or rather an operation of knotting, which does not go without reminding us of a knot in the strict sense, called a "trefoil knot" (in white in the drawings below).



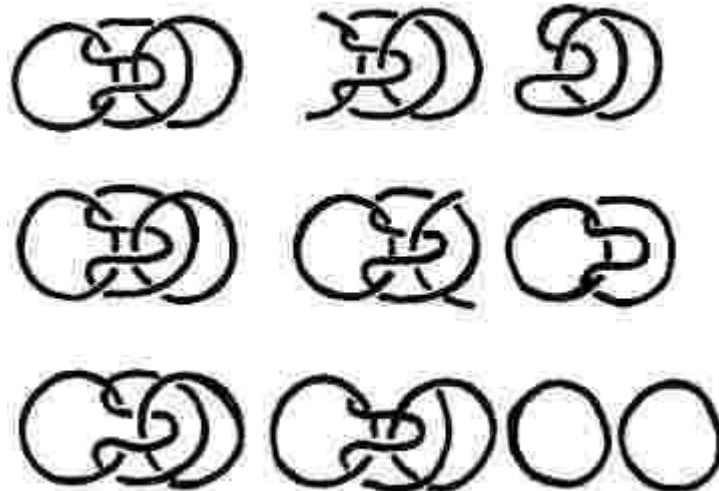
What is the Borromean quality of a chain?

"A Borromean chain is a chain such that if one cuts any one of the rings, all are unlinked." One can represent the chain in a way that puts the accent on the possibility of multiplying to infinity the number of distinct threads:



In this representation, called "a generalized Borromean chain," it is clear that the central loop, in the form of a crescent, can be multiplied: the number is not relevant to the Borromean quality of the knot. It suffices to cut any one among them to undo the knot.

The following drawings show how a Borromean knot of this type is undone, whichever loop is cut:

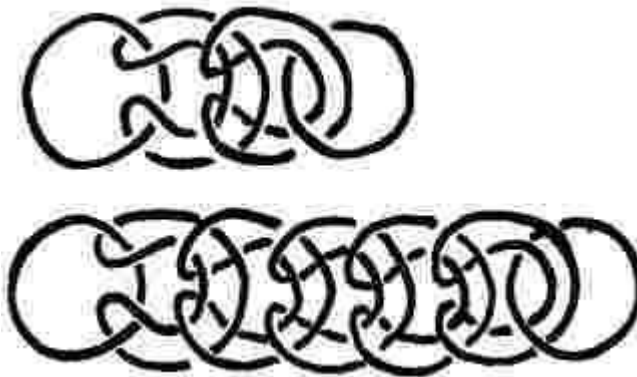


There are three ways of cutting any Borromean knot, and they introduce two aspects essential to the Borromean knot: the number three and the putting flat (which is to say, the manner of representing a knot or a chain by a drawing).

For the representation of the generalized chain, there are three different drawn forms (*tracés*) found regardless of the number of loops one adds at the center:

Another representation exists, the one used most often by Lacan, which presents in its drawn form itself the identical function of all of the rounds, as regards the Borromean quality.

The three rounds of thread each plays the same role *vis-à-vis* the other two: two loops are posed one over the other, and the third ties them together, all three, in a Borromean manner. It suffices, in fact, to cut any one of them for the whole thing to be undone.



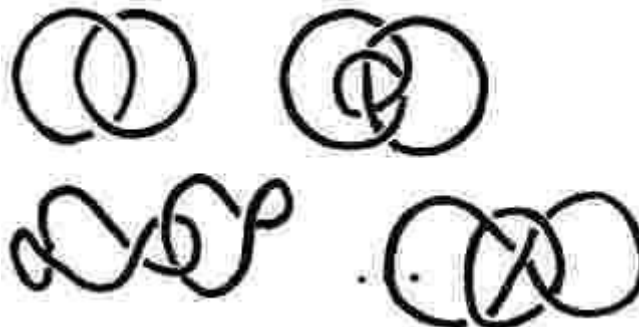
The following drawing shows how a single round (in white) links the two others by an alternation of above-beneaths:



It is in the name of this homogeneity of functions in the drawing that Pierre Soury preserved for the three the role of unity in the classification of Borromean chains. The more specifically mathematical concern of his course of 1980 was "to demonstrate the exemplarity of the three looped Borromean in the classification of chains." He added that the Borromean knot played a central organizing role in the Milnorian classification of chains. This tells us how important this three-ringed knot is.

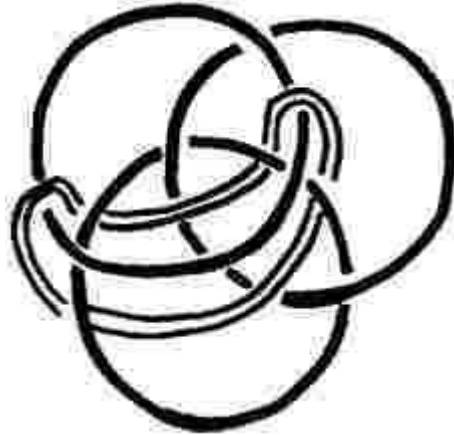
Beginning at three, it should be recalled, the Borromean quality becomes pertinent.

If we knot together, in any manner, only two rings, the Borromean quality is in every way verifiable. This is what logicians call the trivial: however one approaches it, it is verified:

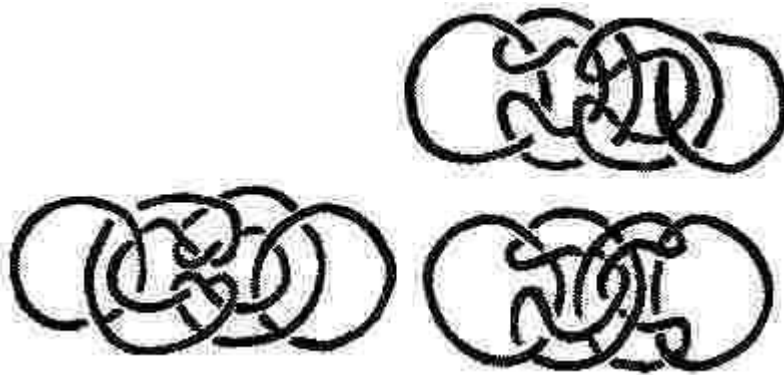


On the other hand, in a Borromean chain with four rings, subgroups always appear, whether one and three, or two and two, as is shown in the following drawings.

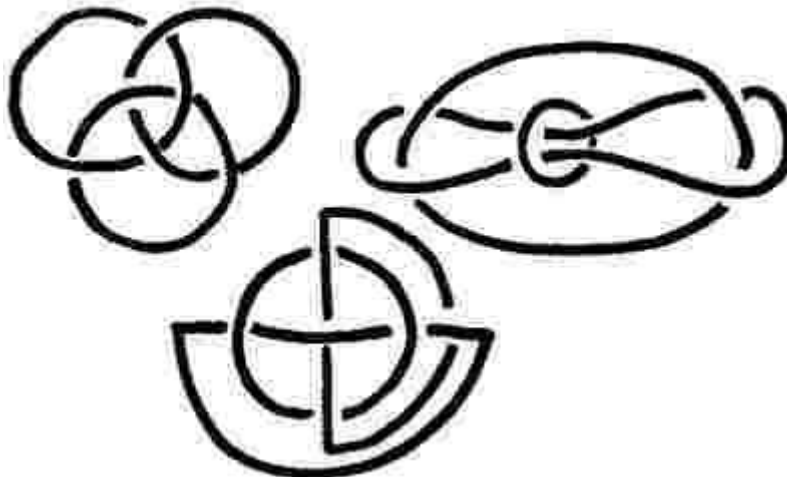
Subgroups, one and three:
(the subgroup of one is in white)



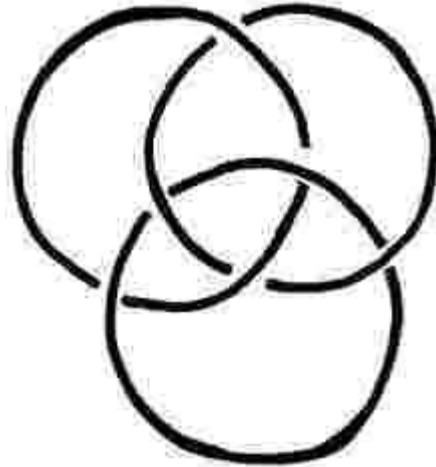
Subgroups, two and two:



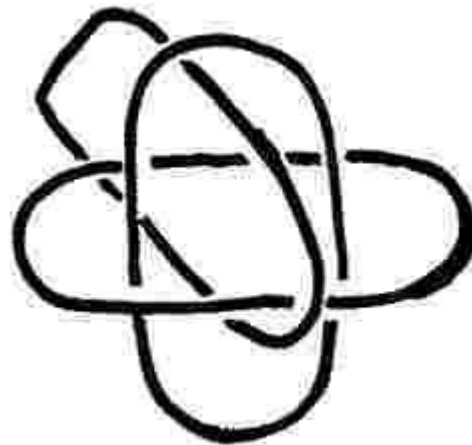
There are many ways to present a Borromean knot:



This presentation, classic in the realm of analytic discourse, has the interest of showing the functional equivalence of the rounds to one another: it writes the homogeneity of three consistencies:



The armillary presentation, on the other hand, called this because it evokes the presentation of the moon and the stars by the armillary spheres of the middle ages, creates a distinction between the knot and the and the schemas that designate sets having intersections and unions:



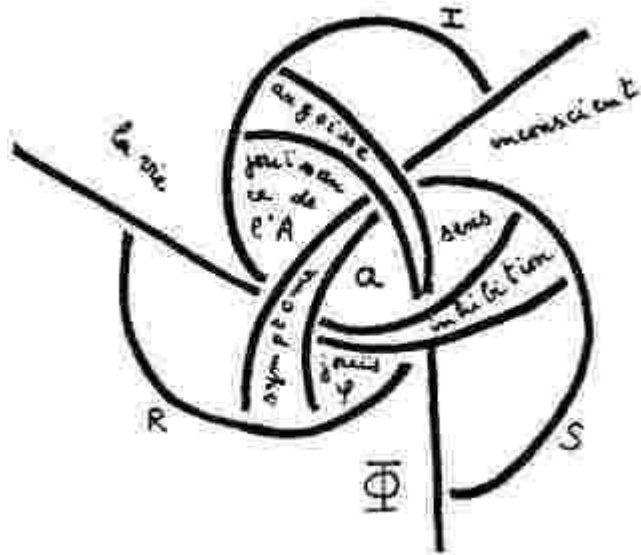
We often forget the above-beneath is not a point of intersection between two lines as in the Venn-Euler diagrams. For chains and knots, this above-beneath is the letter of the imaginary dimension.

We must add that in the domain of the topology of chains and knots a round can always be represented by an infinite line: a circle can, by convention, close itself at infinity . . . Whence the many images of the Borromean knot that Lacan finds in iconographies throughout time:



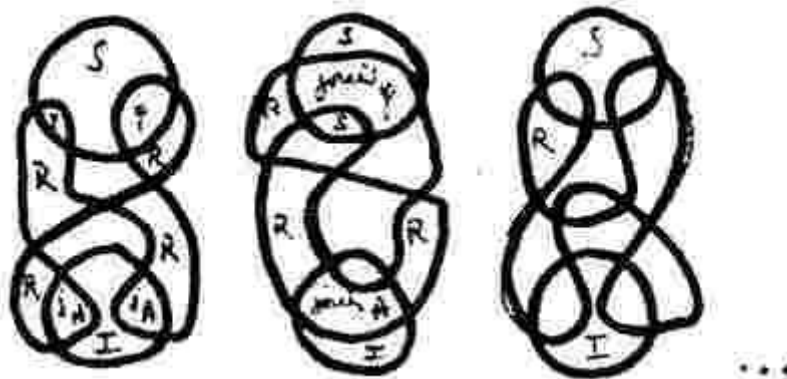
Lacan makes a particular use of this possibility, since he makes of it the writing of existence. But that introduces us into a more complex reflection, which we will attempt to clarify (*mettre á plat*).

The analytic use that Lacan makes of the Borromean chain is more explicit in the Seminar *RSI*, if only because a drawing supports it. The trilogy of the real, the symbolic, and the imaginary is taken up again by Lacan throughout this seminar and related to the Borromean knot. Though Lacan speaks of chance in evoking the coming into play of the Borromean knot in his reflections, this knot nonetheless acquires a necessity therein that must be commented on. The Borromean knot, or the three-ringed Borromean chain, comes to write the relations exchanged between the three registers of the real, the symbolic, and the imaginary. They write its "common measure."



This drawing is neither a graphic representation, nor a schema, but a topological writing. Just as with a writing, this drawing makes consist, and exist, what is at issue in analytic practice.

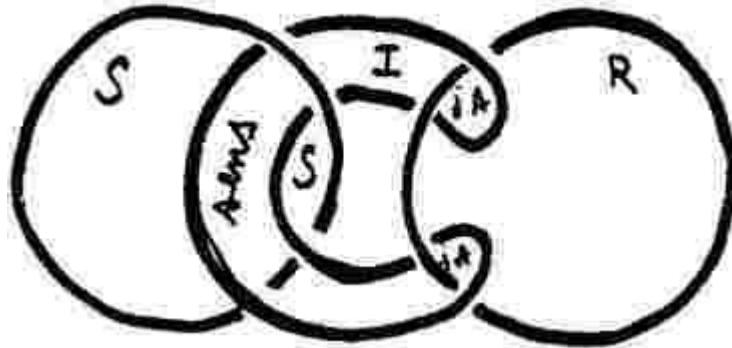
Lacan presented this drawing beginning with his *Séminaire* of September 17. It is at first approach astonishing, and retains all of the piquancy (*sel*) of this astonishment, especially in the spaces delimited by the lines of the drawing. Indeed, at issue is neither an intersection, nor a surface; there is nothing delimited or even measurable about these spaces. They can vary to infinity without transforming the structure of their relation. In his course, Pierre Soury takes on the question of the legitimacy of such a writing with the help of the following designs:



The absence of certain terms is revealed to be symptomatic of a given organization. For example, the drawing evincing courtly love does not include the term "phallic jouissance":

Courtly love is a vision of the world organized around the structural refusal of the sexual realization of love. The absence of phallic jouissance in this form of the Borromean knot results

in the complete masking of the "object (a)." The imaginary, which is in a median position, is the operator of the knotting: courtly love is literary figure, a poetics.¹



The question of how to qualify the simultaneous writing, if we dare this definition, of the little letters, in the interstices of the Borromean knot, still remains mysterious. The drawing is suggestive enough, in its continuous transformations, for having marked the places of absence, of voids. At issue is a way of clarifying the clinic in its infinite variety; is it a question of illustrating what in structure usually remains invisible despite its organizing role?

Lacan allows us to glimpse the direction (*sens*) of his development, although he counsils us to "use it stupidly." He writes: "I have found only one way to give a common measure to the terms Real, Symbolic, and Imaginary, which is to knot them together in a Borromean knot." He adds later: "I have been captured by the Borromean knot," or again:

"The three rounds came to me like a ring to a finger"; "I have always known that the knot incited me to announce of the symbolic, the imaginary, and the real something that homogenized them." It is only a question of finding out how to count them, from the moment that the count begins at three.

Beyond the surprise that such comments can give rise to, we should keep in mind that Lacan demands of the Borromean knot to explain, to formalize, some relations that are not written elsewhere. The knot does not illustrate the relations between the terms, it creates them. To pose the question, or stop at the imprecision of what the words write in the schema recalled above, does not take into account the creative aspect of Lacan's development. The effort of this seminar *RSI* is to say, name, write, and create the words appropriate for speaking of the relations entertained between the symbolic, the real, and the imaginary. In this attempt, Lacan calls on us to re-situate the terms of Freudian research as points of illumination that should help us conceptualize this writing. The written words then find, by their placement between the above-beneaths of the Borromean knot, a new structural dimension: "one triplicity is doubled by another triplicity."

Let us return first to the homogenization between the three registers created by the Borromean knot. In their knotting, the real, the symbolic, and the imaginary have the same function: they can be counted as three. Until then, in fact, there is no necessity of naming only three registers, rather than four or only two. The Borromean knot brings in this necessity. Indeed, Lacan says that other dimensions can be invented; this does not prevent the Borromean knot from always bearing the mark of the three.

¹ Seminar *RSI*, published in *Ornicar*, #2 to 5, *Le Seuil*, 1974-75. This commentary takes all of its citations from these four issues.

He gives to each round, the unity, the "one" that is the "common measure." He gives as three "ones" the real, the symbolic, and the imaginary, which nonetheless are only to be understood in their relations, their knotting.

Lacan seeks proofs for the existence of the Borromean chain as a foundation of thought and above all of sense. It is a question of showing that three is the necessary figure for posing an existence that does not make an image . . . As he says throughout his seminar, the words are contaminated by the reflection made on them. Henceforth existence and image are terms echoing what is different in the three registers.

With the Borromean knot is posed the question of the creation of sense and of its relations with the unconscious and the symptom. We cannot evoke this problem, make sense with these words in speaking of sense and of its birth, without at the same time making intervene what is in question in this research.

We can compare the Lacanian advance on these problems to those problems of nuclear physics where the instrument of measurement, the electron microscope, itself modifies the field of the experience. The examiner only sees the instrument of measurement in these experiences, except for infinitesimal variations revealing the existence of the object of study beyond the instrument.

Thus, Lacan speaks in terms defined by words that measure at the same time their definitions and the gap between the words and this definition: "If I state in speech (*parole*) that the consistency of these three rounds is only supported by the real, it is indeed because I make use of the separation (*écart*) of sense permitted between RSI as individualizing these rounds. The separation is there, supposed taken at a certain maximum. But what is the maximum separation allowed from sense?" These formulations give us the impression of a thought that chases its own tail, that does not situate its object, and that even loses its words and its sense. Beginning in his written introduction added to his oral course, we are given this impression of "futility" or "debility": "It is the type of problem I encounter at every step, without looking for it, I must say, but the measure itself of the effects I say cannot fail to modulate my statement (*dire*). If we add the fatigue involved in this statement, we are still not relieved of the duty to account for it, on the contrary. A marginal note may be necessary to complete the circuit elided in the seminar; it is not the touching up that is 'futile' here, but, as I stress, the mental itself, to the extent that it exists."

How does Lacan define the relations between the three rounds?

Let us try to summarize the diverse definitions given in the Seminar. It is a matter of pinpointing the relations entertained between the real, the symbolic, and the imaginary, which are defined respectively by existence, the hole, and consistency.

The three terms R, S, and I hold together, make three, by the adjunction of the imaginary to two others, "for the triad of the real, the symbolic, and the imaginary only exists by the addition of the imaginary as third." This is why it is appropriate to begin by taking up the term consistency.

Consistency, which Lacan indicates to be equivalent to the imaginary, is necessary for the knot to be, and to be three, since a knot only begins at three. A certain material is necessary, and for Lacan the material is imaginary, inasmuch as it is pegged to the body. In Lacan's teaching, the imaginary refers to the problematic of the image in the mirror. Lacan amply developed the structuring function for the subject of the appearance of its image in the mirror, the image of a little other with which the child identifies in a "precipitation" signaling its entry into the

symbolic. At issue is a knotting of three registers. In this image, the child recognizes the object of the desire of his mother. He identifies with it, and clothes himself with it, and by this means, brings into consistency the symbolic hole, equivalent to the lack presented by the gaze of the mother.

In this way the knotted three is made to consist: "consistency, to designate it by its name, I mean by its correspondence, is of the imaginary order." Each of these three rounds, by their knotting itself, has its own consistency, because thought, sense, is only revealed, is only said, by way of the articulation of three registers. To think the real, there must be a peel of the imaginary. Lacan speaks of the cord: "They have a consistency that I am indeed forced to call real, and which is that of the cord."

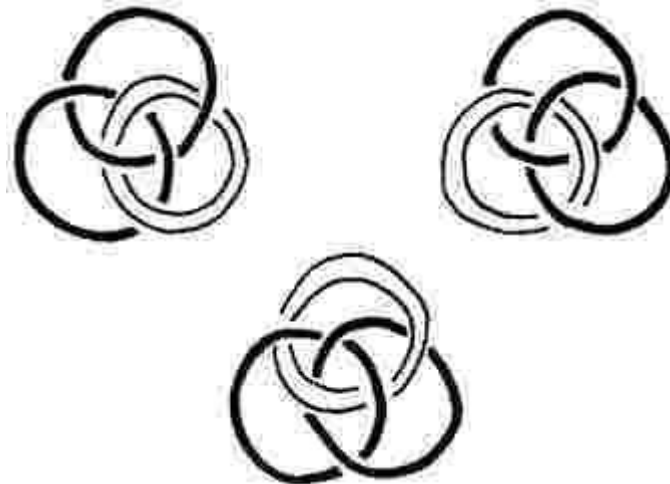
To say the symbolic, the recourse to the imaginary is just as necessary. Lacan then refers to what is implied by the organizing term. In this perspective, what Freud calls the "dead father" is an imaginization of the symbolic.

Just as Lacan says "real to the second power," we must say "imaginary to the second power" to evoke the consistency of the imaginary: "consistency for the speakingbeing (*parlêtre*) is what is fabricated and invented. In this case, it is the knot inasmuch as one weaves it together, but it is precisely not inasmuch as one has woven it together that it exists. This existence is what corresponds to the real."

Existence defines another aspect of the relation between the three rounds.

At the moment of knotting, the consistency, the material of the three rounds must necessarily enter or exit a hole. We will come back to this.

In a Borromean chain, two rounds are posed one over another, without relation, free from each other, and a third knots them: there is always for two of the three rounds a third that realizes the knotting (in white in the following drawings):



Lacan defines this third as existing functionally to the two others. Ex-sist means, more precisely, situating itself elsewhere, although a presence is necessary to the two others as a point of support, of limit, of knotting.

Lacan, in *L'Etourdit*,² shows the necessity of a point of exclusion for sense. The universal only poses itself from a point that it excludes: "there is no universal that must not be contained by an existence that denies it."

² *L'Etourdit*, in *Scilicet 4, Le Seuil* 1973.

We are again at the heart of the problem of sense. One of the functions of the Borromean knot is to show how what is excluded is necessary, or how the tie is made by a third, beginning with the two that are not knotted.

"The existent is what turns around the consistent and makes an interval."

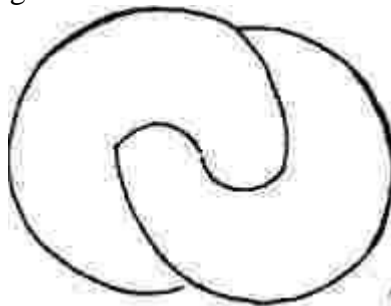
Lacan makes the term existence correspond to the register of the real. By definition, the real is what is not symbolized, what is outside of what makes sense. However, he does pinpoint what is conceivable: "Existence is only defined in effacing all sense."

In the schema in the first seminar of 1974, Lacan situates existence in each consistency. In their drawings themselves, he shows that the function of a round is necessary to the knotting of the two others: Lacan draws, parallel to each round, an open line that he defines as follows: "I propose to symbolize by an intermediary field what exists to the real of the hole; this intermediary field is given by the opening of the round as an infinite line isolated from its consistency."

As for the notion of the hole, its equivalence varies in the course of the seminar: it is at first real, then symbolic. Lacan explains that the Borromean knot allows us to distinguish the hole from existence: existence is made of this infinite line that knots two other rounds. "One of the consistencies is not knotted to the other, does make a chain with it," whence the existence of a third and the non-reciprocity of the passage of one of these consistencies into the hole offered to it by the other. For each round, there is the necessity of a hole in its consistency to permit the knotting, but this hole is differentiated precisely from the third that enters and exits it as the "operator of knotting."

At the beginning of this Seminar, the distinction is not made and Lacan situates the hole on the side of the real: "Whence the correspondence with the hole that I try at first with a real that will later find itself conditioned by existence." At this moment, in fact, he seeks the definition of what is not consistency in the rounds and that allows for their knotting. Later, he says: "We are lead to pose that the hole is of the symbolic order, which I have founded on the signifier." But to think this hole there has to be a peel of the imaginary, or at least the elements of an image that allow us to elaborate it: the topology of the torus then imposes itself.

The torus is a holed sphere, a transformation that encounters the obstacle of another cord supposed to consist. The central hole of the torus is imagined starting with the knotting of another torus in this hole: the image of linked torii:



This image supports the metaphor of the hole. It brings a consistency supporting the contradiction of the not having: the hole has a consistency that is not imaginable. The edge of the hole is imposed as the representation of the hole itself.

"It suffices to imagine the circle as a consistent cord to see that the within and the without are exactly the same thing. There is only a single within; it is what we imagine as the inside of the torus, but the introduction of the figure of the torus consists precisely in our not taking count of it". The torus is a surface without a hole, without a rupture, around a central hole.

As for the hole itself, "no one knows that it is a hole," Lacan adds. Nonetheless, analytic thought puts the accent on the hole, "although this accent plugs it up." Well before its last topological developments, Lacan spoke of the object (a) as the stopper in a hoop net . . .

Topology is the only way to approach this question of the hole and leads to some notions that are not simple. From the start, there is a structural, essential, difficulty to topology. Lacan even speaks of an aversion to it, reading there the trace of an original repression. "Why not see in the aversion that it manifests the trace of primary repression itself?" This is a way to fold the unconscious over the symbolic, and, more precisely, thanks to the reading permitted by the Borromean knot, of making the unconscious "what exists to the symbolic." We can then differentiate what constitutes the hole, primary repression, of which we will never know anything. This original repression, to extend the metaphor, is the symbolic at the second power, the symbolic hole in the symbolic. Death refers to the hole in the imaginary. And all of the orifices in the body, inasmuch as they are interchangeable, constitute what makes a hole in the real: "the inner-tube torus" (*tore-boyau*).

This is how the relations or non-relations between these three rounds of the Borromean knot are summed up. Consistency, as imaginary, existence, as real, and the hole as equivalent to the symbolic form three terms that we must only use in remembering all of the seminar *RSI*. Freudian concepts can now be taken up again at the interior of this writing.

For instance, the unconscious can now be situated in relation to the symbolic, the imaginary, and the real as what exists to the symbolic, which is necessary for the knotting of the real and the imaginary. Likewise, the "phallus" is necessary as "existent, to the knotting of the symbolic and the imaginary, testified to by 'sense.'" There is nothing astonishing about all sense being able to be definitively lead back to phallic signification. "The phallus" is its support, as existent . . .

"Phallus" and "Unconscious" are Freudian notions, derived from analytic experience. They find a place in this writing that transforms the enumeration of their definitions into a structural positioning.

In conclusion, we should hold on to the idea that the Borromean writing of Freudian notions allows for a simultaneous, paradoxical, and illuminating reading, in relation to what can be heard, as the unfolding in the metonymic duration of a discourse, by a psychoanalyst in his armchair.

The Borromean knot offers us a support that is neither a model, nor an illustration, of how the truth is wedged, suspended, at variable points of which the place can nonetheless always be marked (*est pourtant toujours repérable*).