The number thirteen and the logical form of suspicion

By Jacques Lacan

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Plus inaccessible à nos yeux, faits pour
les signes du changeur…
(Discours sur la causalité psychique.)

[85] Once more, we shall begin with one of those arithmetical problems in which the moderns see little more than recreation, not without being tormented by the idea of the creative virtualities which the traditional thought would come to discover in it.

This one is owed to Mr. Le Lionnais, who, we were told, is a grand initiate in these arcana and who, therefore, perturbed the sleep of some Parisians. It was through this prism, at least , that it was proposed to us by Raymond Queneau, who, being a great specialist in the games in which he finds no obstacle in putting to test his dialectical agility, and no less erudite in the publications reserved to their cultivation, can be followed when he affirms that its data is original. Lo:

The problem of the twelve pieces

Amongst twelve pieces of similar appearance, one, which we shall call bad, distinguishes itself by a difference in weight, imperceptible without a measuring apparatus, a difference of which *it is not said if it is of more or less weight.*

We are solicited to find this piece amongst the others, in a total of three weighings, having as our instrument a scale with two plates, excluding any weight of serving us as a standard or any other gadget other than the pieces in question.

The scale which is provided to us as an apparatus will function, for us, as the support of a logical form, to which we refer as the form [86] of ambiguous suspicion, and the weighing will show us its function in thought.*

*Footnote to this section: The study developed here situates itself within the initial formal analyses of a *collective logic*, to which it was already referred in a text published in the previous number of the 'Cahiers D'Art', under the title of 'Logical Time and the Assertion of Anticipated Certainty'.
The form developed here, although it compares succession, is not of the order of logical time and situates itself as being prior to it in our development.
It is part of our exemplary approaches to the conception of logical forms in which the relations of the individual to the collection must be defined, before a class is constituted, that is, before the individual is specified.
This conception is developed within a logic of the subject which our other study allows us to distinctively discern, given that, at the end of the text, we have even tried to formulate the subjective syllogism through which the subject of existence assimilates himself to the essence, which is, for us, radically cultural, to which one applies the term humanity.

Solution of the problem

This problem requires an operational invention of the utmost simplicity and which is totally within the reach of human spirit. We doubt, nevertheless, that it is within the reach of the mechanics whose wonder the name "thinking machine" expresses very well. There would be much to say about the order of the difficulties opposed to spirit, respectively, by the forms developed in the game of numbers and by the simplest forms in which the question is to know if one contains implicitly the other.

Thus, to whoever wants to experiment the resolution of our problem, let us clarify here that its conditions must be rigorously accepted - that is, that any result that is found, when one puts 2 pieces or 2 sets of pieces (always, evidently, in equal number), will count as one weighing, no matter if the plates remain balanced or if one of them prevails.

This observation aims at making sure that the enquirer, when he finds himself in the apparently inevitable moment in which the difficulty will present itself, do not tergiversate, presuming, for example, that a double attempt, referring himself to the same operational time, can be counted as only one weighing, but that, rather, animated by the certainty that there is a solution, persevere at the depths of the impasse until he finds its flaw. Let him join us then to consider with us its structure. [87] Let us guide, for now, the more docile reader.

The small amount of proofs allowed orders that one should proceed in groups. The remembering of the assured presence of the bad piece amongst the 12 could dissuade us from initially dividing them in half between the plates: this remembrance, in effect, making it certain that one of the groups of 6 will weigh more than the other, could lower our interest in such a proof. But this reasoning will reveal itself to be merely approximative.

The true justification of the successful procedure is that the weighing in a scale with two plates has three possible results, according to their equilibrium or to the prevailing of one over the other. It is true that, in the case of their unbalance, nothing reveals to us on which side is the object which is responsible for this unevenness. Still, we have legitimate reasons to operate according to a tripartite distribution, a form whose incidence we find more than once in the logic of collection.

The first weighing and the problem of the four

Taken from the set of 12 pieces, therefore, let us place in the scale two groups of four.

The situation of equilibrium between them allows us to locate the bad piece amongst the other four. A problem whose solution seems easy in two further weighings, though it is convenient to formulate it without precipitation.

Let us clarify that, in the second weighing, we shall place in each plate one and only one piece. Did the plates remain in balance? In this case, the two pieces are good, and one of them, opposed to one of the remaining ones, in a third weighing, will either bring into evidence the bad piece or will allow us to situate it, through elimination, as the last one which was not tested.

One of the plates gets, on the contrary, more heavy in the second weighing? The bad piece will be amongst the two sets in the scale, so that the remaining two pieces are surely good, the situation, similar to the one of the previous case, will be solved in the same manner, that is, comparing between them one piece of each group.

The development of the problem will show that it is not superfluous to signalizes here that this procedure solves a problem that can be considered autonomous: [88] that of the bad piece being detectable between the four, through two weighings, that is, the problem immediately inferior to ours. The eight pieces implicated in our first weighing did not intervene at all, in effect, in the search for the bad piece amongst the other four.

The x of the difficulty and the divided suspicion

Let us return now to this first weighing to consider the case in which one of the two groups of four pieces in the scale is heavier.

This case is the x of the problem. Apparently, it leaves us to detect the bad piece amongst the eight ones and it leaves us to do so in two weighings, after these two weighings having shown themselves sufficient to detect the piece amongst four ones.

But though the bad piece remains to be recognized between the eight, the *suspicion*, shall we say, that falls upon each one of them soon becomes *divided*. And here we touch upon an essential dialectics of the relation between the individual and the collection, insofar as they comport the ambiguity of the more or the less.

Therefore, the result of the second weighing can be formulated as follows:

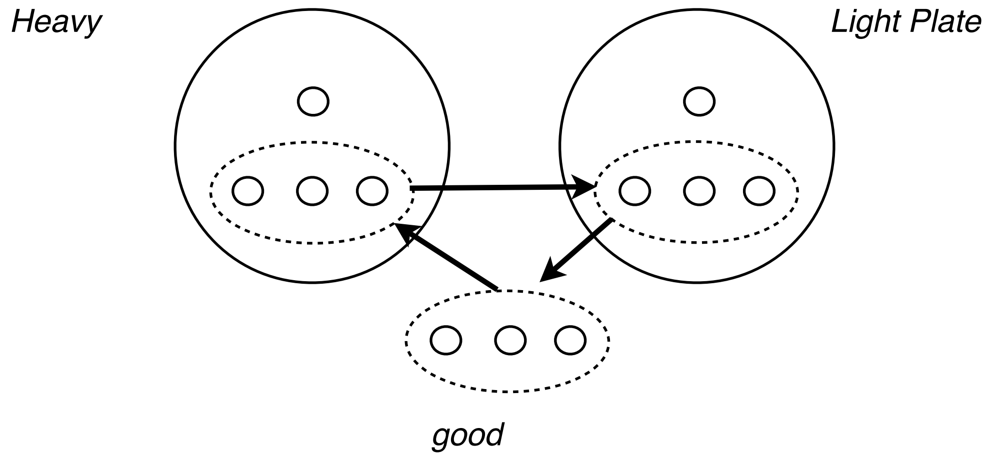
The pieces which are on the heaviest plate are only suspected of being heavier; those which are only suspected of being too light.

The tripartite rotation or 'the tri'

SUch is the root of the operation which allows us to solve our problem, and which we shall call the *tripartite rotation*, or in a pun with its function of screening [triage], the *tri.*

This operation will seem to us the knot in the development of a drama, be it the problem of the twelve, be it, as we will see, in its application to superior collections. Here, the third weighing, just as, in other cases, in all the following weighings, will be figured by it solely as a liquidating outcome.

Here is the scheme of this operation [89]:



The tripartite rotation

We see that three pieces, already determined as good, were made to intervene, just as in fact they were provided to us, another result of the first weighing, within the four remaining pieces - since the bad piece is certainly within the eight included in the weighing.

There exists, on the other hand, a form of this operation which does not make these pieces intervene - and proceeds by the re-distribution only of the pieces already in the scale, after the exclusion of some of them. But, whatever the elegance of this economy of elements might be, I shall keep to the exposition of the form represented above - for several reasons:

1st) because the tripartite distribution of the elements in the test which immediately precedes the operation necessarily provides a number of elements which, purified of suspicion, are always more than sufficient so that this form can be applicable in the extension *ad indefinitum* which we shall give our problem and, even further, as we shall see, in the essential complement that we shall bring to it.

2nd) because this form of the operation is more manageable mentally by those who are are yet accustomed to conceive it by subjecting themselves to the proofs of their findings.

3rd) because, lastly, once resolved by the weighing that concludes it, it is the one which leaves less complexity to the liquidating operations.

[90] Our tripartite rotation consists, thus, of the following:

Of putting three good pieces in the place of three indistinct pieces of the heavier plate, for example, and then using the three extracted pieces of this plate to substitute three pieces removed from the lighter plate, which, from then on, will be removed from both plates.

The second weighing and the decisive disjunction

One have only to realize, in a second weighing, the effect of this new distribution in order to conclude, according to each one of the three possible cases, on the following results:

First case: the plates are balanced. All the pieces are therefore good ones. The bad one will be found, in this case, *amongst the three excluded pieces* from the plate which showed itself lighter in the first weighing and, as such, we know that it can only be *a piece lighter than the others.*

Second case: shift of which plate is the heaviest. In this situation, the bad piece has changed plates. It should be found, therefore, *amongst the three pieces which left the plate that revealed itself the heaviest in the first weighing.* As such, we know that it can only be *a piece heavier than the others*.

Third case: the balance remains uneven as it was after the first weighing. In this case, the bad piece *remains amongst the two which were not moved*. And we also know that, if it is the piece that remained in the heaviest plate, it is *a heavier piece*, and if it is the other, then it must be a piece *lighter than others*.

The third weighing in the three cases

Taken to this degree of disjunction, the problem no longer offers a serious resistance.

In effect, a piece, which has already been determined as being lighter, in one case, heavier, in the other, will be identified amongst three, in a weighing that will place in the plates two of them, and in which it shall appear without ambiguity; otherwise, it will reveal itself to be the third one.

[91] Regarding the third case, we have only to bring together the two suspected pieces within the same plate and put on the other plate any two good pieces, which are already beyond suspicion, so that the weighing will present to us the bad piece. Indeed, the plate with the suspected pieces will manifest itself, either as heavier or as lighter than the other, for it surely contains a piece that is either too heavy or too light, so we will know which one of them to incriminate, even if we have lost from sight the individuality of each of them, or, to

put it differently, from which plate of the second weighing it came from.

And thus is the problem solved.

The maximal collection accessible to n weighings

Could we deduce, from this, the rule that, once determined the number of weighings, would give us the maximal number of pieces amongst which these weighings would allow us to detect one, and only one, characterized by an ambiguous difference - in other words, the ration of the series of maximal collections determined by an increasing acceptance of weighings?

In effect, we can see that, if two weighings are necessary to identify the bad piece in a collection of four, and if three allow us to solve the problem of the twelve pieces, this is because the two weighings remain enough to discover the piece amongst eight, given that a first weighing divided the halves between which the suspicion of excess and lack is divided. We shall easily confirm that an adequate application of the tripartite rotation allows us to extend this rule to superior collections, and that four weighings could easily solve the problem of 36 pieces, and so, successively, multiplying by 3 the number N of pieces, every time we attribute one more unity to the number n of allowed weighings.

Formulating N as equal to 4 times 3^{n-2}, could we determinate the maximal number of accessible pieces to the depuration of n weighings? It is enough to attempt this to realize that the number, in fact, is bigger, and that the reason for this is already patent at the level of our problem [92].

Mr. Le Lionnais, either because he obeyed the traditional precept which orders that, when one knows ten things, one should only teach nine, or because of his benevolence or malice, shows us having made things too easy for us.

Though his data brought us, indeed, to a procedure which conserves its value, we shall see that the comprehension of the problem would be mutilated to those who did not perceive that three weighings are capable of detecting the bad piece not only amongst twelve pieces, *but amongst thirteen.*

We shall demonstrate it now.

The problem of the thirteen

The first eight pieces represent well everything that can be put into play in the first weighing. And, in the eventuality of them all being good pieces, a case which we have already contemplated above, there will be five pieces left, amongst which two weighings seem insufficient to determine the bad piece, and it would surely be insufficient, if, at this level of the problem, these five pieces were all that we could use.

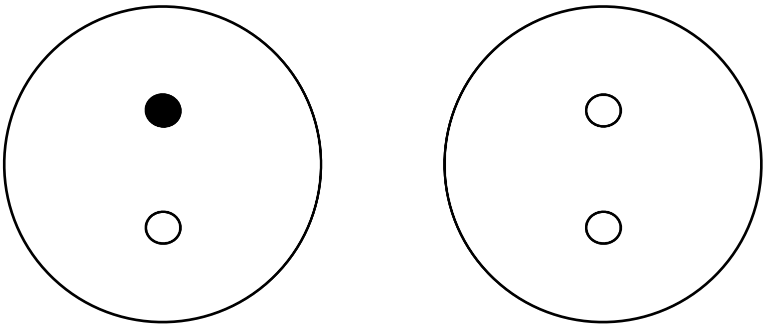
Indeed, when we examine the problem limited only to two weighings, it becomes clear that four pieces are the maximal number accessible to them. We could also observe that only three pieces can be effectively put to test there, a fourth one never being placed in one of the plates and only being incriminated, in the extreme case, by the data which attests to the existence of a bad piece.

The same observation is valid for the group which we are considering as the residuum of the superior problem (and it shall be valid only for this one case, because the detection of a piece by elimination, through a weighing that it is not involved, as we have observed in other possible moments of the problem, springs from the fact of its presence in the group having manifested itself in a previous weighing.)

But, when our group of five pieces is presented as a left-over, the case is not similar to the one of the isolated four pieces. Here, other pieces, through a previous weighing, have been recognized as good, and only one is enough to modify the reach of the next two weighings that have been conceded to us.

The 'by-three-and-one' position

Indeed, let us dignify the consideration of the following figure [93]:



The 'by-three-and-one' position

We will admit to recognize there the two plates of the scale, there being, on one of them, in the form of a filled circle, the good piece which we shall introduce in this plate together with one of the suspected pieces, and, on the other, a pair of these five suspected pieces. Such will be the disposition of our second weighing.

Two cases:

Either one of the plates will prevail, and we will realize that the suspicion is divided, but, here, in an uneven way: between a sole piece, suspected in one way, and two, suspected in the inverse way.

It would be enough then that we take one of the two remaining pieces, from now on guaranteed as good pieces, and substitute it for the isolated suspected piece, and substitute one of the other suspected pieces from the other plate for this removed piece, executing the utmost reduced of the tripartite rotations, the triple rotation, so that the results become immediately visible in the third weighing:

- either the same plate will prevail, making it evident that the bad piece is the one of the pair of suspected pieces which was not moved.
- or there will be balance, showing that the bad piece is the other one from that same pair, the one that was expelled from the plate. [94]
- or, in the case the prevalence alternates, the bad piece will be the isolated one which changed plates.

The decisive disposition here, the one which orders the weighings of the three suspected pieces together with a good piece, we designate it as a ‘by-three-and-one’ position.

This ‘by-three-and-one’ position is the original form of the logic of suspicion. We would make a mistake if we were to confuse it with the tripartite rotation, though it finds its solution through that operation. On the contrary, we can see that only this position gives the operation its full efficacy in our problem. And, in the same way as it appears as the true resource in its solution, only it allows for the revelation of its authentic sense. This is what we shall demonstrate now.

The problem of the forty

Let us move now to the problem of four weighings, to find out up to which number of pieces its reach extends itself, keeping to the conditions of the problem.

Quickly we realize that a first weighing can involve, with success, not only two times twelve pieces, but, according to the rule suggested by the first resolution of the so-called problem of the twelve, also two times thirteen pieces.

Indeed, in the appearance of the unbalance, the operation of the tripartite rotation, using nine good pieces, is capable of detecting amongst the 26 pieces of the first weighing the bad piece in another three weighings.

The weighing after the *tri* will separate the pieces into two groups of nine pieces of univocal suspicion, in which case a third weighing of three against three will shed light on the bad piece, be it in one of these groups, be it in the three remaining pieces, or, in any case, it will be isolated by a fourth and last weighing, and, in a group of eight, of divided suspicion, in which we already know how to find the piece in two weighings.

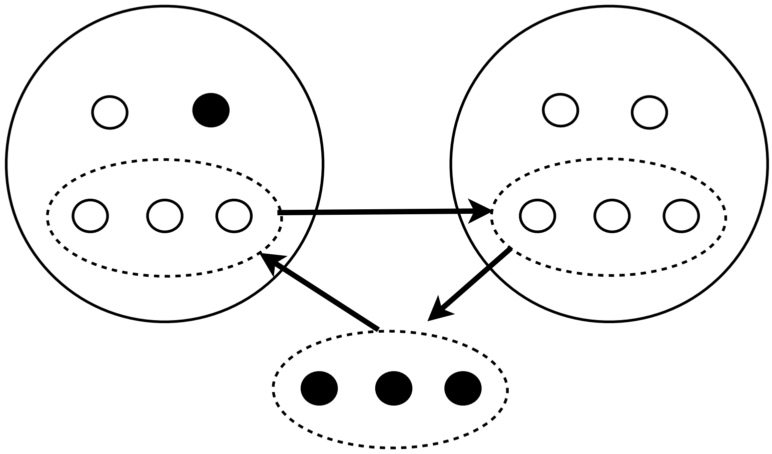
But, having revealed themselves good all the 26 first pieces, we will be left with three weighings, and that is when the 'by-three-and-one' position will demonstrate its value.

[95] To occupy the field with a new *tri*, it will indicate us, indeed, that we should put into play not only four pieces against four, as the study of the case of the three weighings suggests, but five pieces against four pieces complemented by a good piece. After the preceding demonstrations, the following figure will be enough in the demonstration of the possibility of solving the position of the nine pieces, when the bad piece is revealed by the unbalance of the two plates.

Next we see the *tri* scheme, which, in the proof of the third weighing, will reveal in which group of three suspected pieces the bad piece is, being enough to have a fourth one to isolate it in the totality of cases.

But, if the balance of the plates makes it evident that the bad piece is still not there, we who will be reduced, after this, to the margin of two weighings, shall act as in the correspondent level of the problem of thirteen pieces, placing three new suspected pieces on the scale, two against one, with the help of a good piece, and, not witnessing the revelation of the presence of the sought one (which can therefore be isolated in the next weighing), there will be yet one weighing to test yet another piece, until we will manage to designate the bad piece in another, final weighing, solely based on the data that such a piece does exist.

From this it will result that, in the proof of four weighings:
26+9+3+1+1= 40 accessible pieces.



The tri completed in the 'by-three-and-one' position

The general rule of the conduction of operations

[96] If we reproduce the same investigation with a superior number of pieces, we will see the appearance of the rile which orders the conduction of operations of this investigation. It is:

To bring into play the *tri*, if the bad piece reveals its presence amongst the ones involved in the first weighing. If that is not the case:

To introduce the 'by-three-and-one' position, given that we have the access to a good piece, that is, in the conditions here exposed, with the ordering of the second weighing, and to renew it in all the following weighings, until the bad piece reveals its presence in one of them.
To apply then the tripartite rotation, which is the decisive moment of the whole operation. The 'by-three-and-one' position isolates itself in one of the groups, whose disjunction is operated by the *tri*.

If the weighing which concludes this *tri* identifies the piece in the referred group, the only complex case to be solved, then to repeat in it the *tri* with the same possibility that we might maintain the 'by-three-and-one' position and the same indication to resolve it to exhaustion.

Some supplementary rules should be added regarding the condition of investigating any collection whatsoever, that is, a collection that is not maximal.

The ratio of the series of maximal collections

But these rules allow us to see that five weighings can reach, maximally:

1+ 1+ 3+9 +27 +80 = 121 pieces;

- six weighings will reach:

1+ 1+ 3 +9 +27 +81 +242 = 364 pieces (singular number),

[97] and so successively:

- and that, in the algebric form, the true formula of n anteriorly sought will be such that:

$n = 1+ 1+ 3+ 3^2+ 3^3... +(3^{n-1}-1)$

or:

$n = 1+3+ 3^2+ 3^3+...+ (3^{n-1})$,

In which we see that each number N which corresponds to a number n of weighings is obtained through the multiplication of the number N', which corresponds to (n-1) weighings, by three, adding a unity to this product.

This formula expresses in perfect evidence the tripartite power of the scale after the second weighing and, as such, brings to light, by its mere aspect, that the operations were ordered in such a way as to fill all the numeric field offered by this power.

This confirmation is specially important to the first numbers of the series, because they demonstrate their adequacy to the logical form of the weighing and, particularly, to the number thirteen, insofar as the apparent artifice of the operations which made us determine it could leaves us in doubt, be it regarding of a new juncture allowing to surpass it, be it regarding the fact that it leaves empty a fractioned margin in the dependency of some irreducible discontinuity in the arrangement of the operations of dissymmetrical aspect.

The meaning of the number thirteen

Therefore, the number thirteen shows its meaning as expressed in the ‘by-three-and-one’ position - and not, certainly, because it is written with these two figures: this is not more than mere coincidence, for this value pertains to it, independently of its reference in the decimal system. It follows from thirteen representing the collection determined by three weighings, the ‘by-three-and-one’ position demands in its development three proofs: the first, so it can provide the individual purified of suspicion, the second, which divides the suspicion between the individuals that it includes, and a third which discriminates them, after the triple rotation. (This is done differently than in the *tri* operation, which demands only two).

The logical form of suspicion

[98] But, in light of the formula for N, we can advance further in the comprehension of the ‘by-three-and-one’ position and, at the same time, demonstrate that, in our problem, the data, though contingent, is not arbitrary.

If the sense of this problem is related to the logic of collection, in which it manifests the original form which we designated by the name of suspicion, it is because the norm with which the ambiguous difference that it supposes relates itself is not a specified nor a specifying rule, but only a relation of individual to individual within the collection - a reference not to the species, but to uniformity.

This is what is made clear when, remaining given that the individual who carries this ambiguous difference is unique, one suppresses the data of his existence in the collection, so as to substitute him for the concourse of a standard individual, given outside of the collection.

Thus, we can be surprised to realize that rigorously nothing was modified in the forms nor in the numbers to be determinate by the new data applied to our problem.

Here, certainly, the pieces having to be tested up to the last one, none could be taken for the bad one in the position of the residuum of the last weighing, and the reach of this weighing will be diminished by one unit. But the standard-piece, since we can make use of it since the beginning, will allow us to introduce the ‘by-three-and-one’ position since the first weighing and this will augment in one unit the group that is included in it. Well, the datum of this piece, which seems of such great value to our intuition, formed in the classificatory logic, will have absolutely no other effect.

Through this, it becomes evident that the uniformity of the object of our problem's data does not constitute a class, and that *each piece must be weighed individually.*

Indeed, whatever the number of individuals at the cause of our problem, the case demands to be reduced to that which is revealed by the *unique* weighing: to the absolute notion of difference, root of the form of suspicion.

This reference of the individual to each one of the others is the fundamental requirement of the logic of collection, and our example demonstrates that it is far from unthinkable.

The scale of Judgement Day

[99] To express it in the register of a dream which obsesses men, the Judgement Day, we shall indicate that, fixing in the billions the number of beings which this grandiose manifestation would imply, and only being able to conceive its perspective from the soul as unique, the testing of one for all the others, according to the pure ambiguity of the weighing which represents for us the traditional figures, would take effect, with room, in 26 stages, and therefore the ceremony would have no reason to prolong itself for too long time.

We dedicate this apologue to those for whom the synthesis of the particular with the universal had a concrete political sense. As for the others, let them strive in applying to the history of our time the forms we have demonstrated here.

The phenomena of number and the return to logic

By searching again in numbers for a generative function of the phenomena, we seem to return to the ancient speculations whose approximative character led them to be rejected by modern thought. And it seems to us, precisely, that the time has come to recuperate this phenomenological value, under the condition that its analysis should be pursued in extreme rigor. Probably, singularities will appear which, though they are not without a stylistic analogy to those manifested in physics, or even in painting, or in the new style of chess, will disconcert the spirits, there where their formation is not more than habit, giving them the sensation of a break in harmony that would come to dissolve its principles. If we suggest that one must realize a return to logic, it is to reencounter its basis, solid as a rock, and no less implacable when it enters into movement.

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