

The Klein Bottle

Chapter 5: From the Specular to the Non-Specular

There exists in general topology a phenomenon that, although patent, is still usually misrecognized: the left-handed or right-handed twist in topological objects.

It is no exaggeration to affirm that that science was nonetheless created around this phenomenon. For example, Galileo moves the twist of the sun toward the earth from the exterior to the interior.

Moreover, life displays this twist at key points of its appearance. So it is for the double helix of DNA, which accounts for the physico-chemical structure of chromosomes, but so it also is for the umbilical cord, which is a triple braid (*torsade*) made up of an artery and two veins.

However, as a group, physicians, like mathematicians, care little for this phenomenon; they evacuate it totally from their definitions, an oversight that is no doubt not usually due just to chance.

Indeed, the importance of the mirror and of the reversal it effects in this set of facts is primordial.

For the mirror, the object and its reversed image are identical. The right-handed twist becomes a left-handed twist, but they are perceived as being the same.

Moreover, man finds in the reversed mirror image an illusion so primordial to his identity that this right-left difference remains unrecognized.

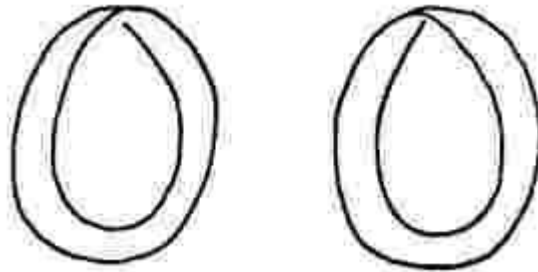
Man is his reversed image, his enantiomorph, his specular image.

Lacan had to come along, with the importance he knew to give to this identification with the image, to undo this feeling of identity between left-handed right-handed twists.

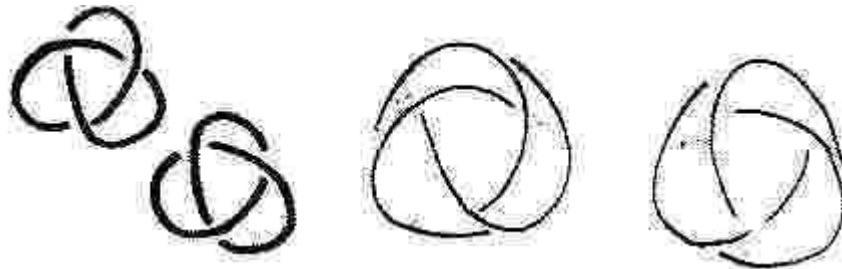
More precisely, it is thanks to the formulation of an object (a), without a specular image, that we can conceptualize (*penser*) the image and its reflection in their originality and in their effects.

Let us clarify the topology of this action (*agencement*): there are images that have an entiomorphic image, which is to say a specular image.

Thus the Moebius strip presents a right-handed or left-handed twist:



In the same manner, there is a right-handed trefoil knot and a left-handed trefoil-knot, depending on whether it is the edge of a Moebius strip with three half-twists to the right or to the left:



When I say "there is," I refer in fact to a topological given: two objects are said to be different on the condition that it is impossible to pass by continuous transformation from one to the other. In this domain, where the forms are as supple as rubber, it is not everyday that we find two forms of which we know with certainty that they are different. Now it is impossible to transform in a continuous fashion a left-handed Moebius strip into a right-handed one. The direction (*sens*) of the twist insures a radically different existence for each.

The mirror inverts the object it mirrors along an axis of vertical symmetry of right and left (and not, for example, of bottom and top . . .)

This letter E becomes this sign \exists ; a right-handed twist becomes left-handed.



On the other hand, whenever the object has a vertical, internal axis, the image in the mirror is identical. The letter **A** becomes **A**; it is not transformed by its mirror reflection.

This is the case for the body of man, which seems to have an axis of vertical symmetry. (Let us remember that this is an illusion of representation, because . . . the heart is to the left, or to the right in the mirror, the appendix to the right . . .).

In these right-handed and left-handed pairs of objects, the symmetry is only apparent and in fact creates an untraversable barrier. Never will a left-handed Moebius strip become a right-handed one. They are totally different, the one from the other, even if they have the same properties. The twist creates a symmetry in relation to the mirror that differentiates them totally.

It is *apropos* of this relation of the object with its image that Lacan finds the imaginary relation constitutive of the Ego (*Moi*). Man looks at himself in the mirror and recognizes himself in the reflection he glimpses.

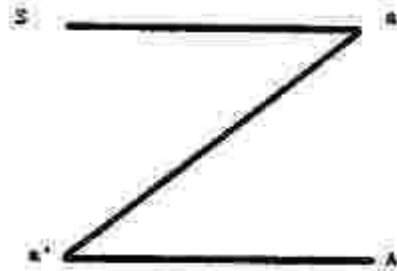
In 1958, in the schema L, Lacan defines the condition of the subject starting from this relation.

Thus: the subject is a participant in this discourse insofar as he is pinned to the four corners of the schema: "(S), his ineffable and stupid existence.

-(a), his objects.

-(a') his ego, or rather what is reflected of its form in the objects.

-and (A), the place from where can be posed for him the question of his existence."¹



This relation between (a), the objects, and (a'), the ego, is the first articulation that Lacan works out concerning the problem that occupies us. We see that the "ego" already has a definition that calls for some developments.

Between (a) and (a'), all of the imaginary relation is in play. At issue is a particular space on which Lacan places his patent. "Imaginary" does not mean false, unless its falsity is structural and necessary to the establishment of the discourse in which the neurotic finds his place. The psychotic suffers from its failure (*carence*), from the non-installation of this imaginary relation.

The mirror stage is in fact fundamental to the installation of the imaginary couple. Man is alienated in his own image, not recognizing the twist.

That identificatory haste (*précipitation*), which, as we know, unleashes a jubilation for the child, has multiple structuring aspects. Its failure allows us to establish some elements of the clinic of the psychoses:

In fact, the unity both of the subject and of the object is structured in the specular relation.

Schizophrenia, with its array of clinical manifestations that can be summed up under the rubric "fragmented body" (*corps morcelé*), is one consequence of the impossible unity of the subject. Different instances cut into the body itself of the psychotic. "The body itself is all-important": it makes use of the (a), the (a'), and the (A), and seeks despite everything to give a consistency to discourse.

The specular relation structures the unity of the object: on this basis, Lacan speaks of the paranoiac structure of knowledge (*connaissance*) . . . : "what constitutes the Ego and its objects

¹ *Ecrits* of J. Lacan, *D'une question préliminaire à tout traitement possible de la psychose*, in *Ecrits*, Seuil, 1966.

under the attributes of permanence and of identity and of substantiality, in brief, in a form of things very different from what we know of the gestalts of the animal world."²

Although it is possible to evoke schizophrenia when this relationship fails, paranoia results from the formalization itself of the unity of the object.

For the paranoiac, (a) and (a') only support their difference on themselves. The subject and his image only find support for their separation on themselves, whence the precipitation of this separation into a mortal rivalry. The object is fixed in an image frozen in the mirror where the subject can only read the aggressivity of a *semblable*, the little other.

His own unifying image is the enemy, whether it is a question of neighbors who wish him ill and spy on him, or if this hatred, supposed in others and misrecognized, throws him into infinite recriminations. Others are depositories by their gaze of this image where he can only be alienated. To separate himself from it, his only possibility is to kill it, to invoke its death, so that he can exist as a subject in his truth as speakingbeing (*vérité de parlêtre*).

Here, death is the equivalent of a call to the symbolic. Lacan formulates it as follows: "the imaginary couple of the mirror stage by what it manifests of the against-nature, if we must relate it to a prematuration of birth specific to man, finds itself appropriated as the base of the imaginary triangle that the symbolic relation can in some fashion cover over."

"It is, in fact, by the gap that this prematuration opens in the imaginary and where the effects of the mirror stage swarm that the human animal is capable of imagining himself mortal."³

Death holds the functional place of the symbolic, because, in this false identity of the object with its image, if the mirror illusion is not sustained, the identity of the one is achieved at the price of the death of the other. Thus, there exists a struggle to the death between man and his double, between man and his shadow, which accounts for his difficulties (*échecs*) with the illusion of identity, such as is produced in the mirror.

The symbolic relation, the possibility of articulating a signifying chain, is supported by an object without a specular image, of which the prototype is the disk bearing the point Φ detached from the cross-cap.

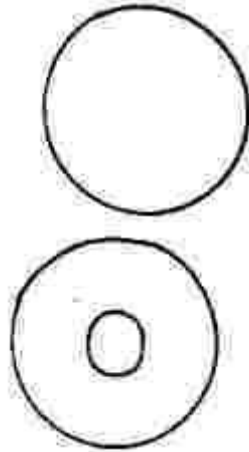
There exist, in fact, objects without a specular image, which Lacan defines with the term object (a): the breast, the feces, the gaze, and the voice.

The cross-cap also accounts for their articulation with the point Φ . Objects without a specular image are images without a double. So it is with the sphere, related by Lacan to the breast,⁴ and the torus, related by Lacan to the *scybale*, the "feces." These are objects whose internal axes of symmetry make it so they are their own inverse; objects somehow coming before the right-left distinction, the creation of the twist.

² Jacques Lacan, *L'Aggressivité en psychanalyse*, in *Ecrits*, Seuil, 1966.

³ *Ecrits* of J. Lacan, *D'une question préliminaire à tout traitement possible de la psychose*, in *Ecrits*, Seuil, 1966.

⁴ *Séminaire du 26 mars 1969*, unpublished. *D'un Autre à l'autre*.



Remember that it is possible to apply a cut to the torus that produces a bilateral Moebius strip with four half-twists, right-handed or left-handed, depending on the trajectory of the cut around the central hole. We thus find the possibility of a gyration, toward the right or the left, which we do not find on the sphere. In a certain differentiation of structures by their gyration, an increasing complication can account for the impression of a progress often ratified as such in the clinic. It is not a question of producing an explanatory principle for this, but of showing how the structures themselves produce this illusion.

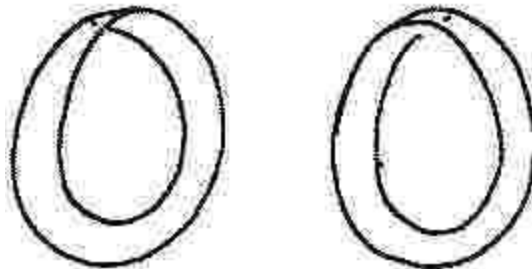
The two other structures on which Lacan supports the two objects (a'), the gaze and the voice, are the cross-cap and the Klein bottle. But these surfaces go beyond the left-right distinction; they envelope it, we could say, not as a being without sex, asexual, but as a being with two sexes. They bring into view the set of the possibilities of gyration to the left and to the right.

The recourse to the Platonic myth of the androgyn is not anodyne. There is, in fact, in these structures a resistance to representation, or an effect of obscenity which is related, existentially speaking, to the fascination or disgust provoked by the evocation of the genital organs, above all in the figure of hermaphroditism.

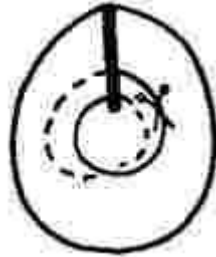
We will see the role of the Φ , the phallic symbol, in this place.



Lacan supports the object (a) that is the gaze with the cross-cap.



The cross-cap is an object without a specular image because the Moebius strip from which it is constructed loses its right-left disparity. As we have seen, the twist is replaced by an intersection, as soon as we fold the Moebius strip lengthwise. It is on the cross-cap, remember, that Lacan supports the cutting of the fantasy that detaches an object.



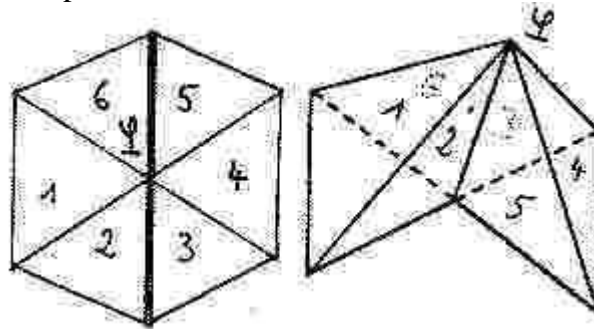
The gaze is an object that falls, disappears as such, in the jubilatory assumption of the mirror stage.

In the same register are situated two points of structure: on the one hand, the words that pertain to thought are in the register of the gaze and vision. On the other hand, the cross-cap allows us to establish the function of the point Φ . For Lacan subsumes all of the objects (a) under the term Φ .

It is not at all surprising that this point Φ , if theoretically establishable in all of the structures, is particularly so beginning with the cross-cap.

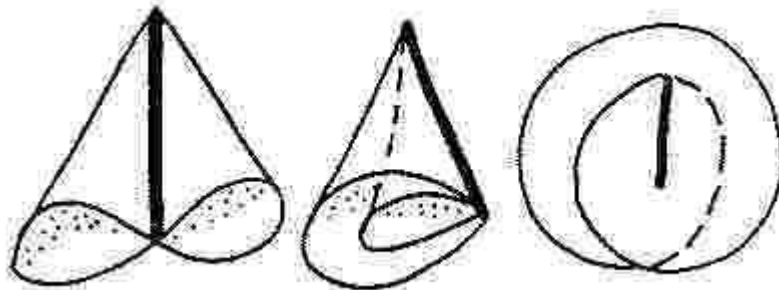
The object (a) in the scopic drive is the most evanescent, which is not unrelated to the place of vision in the mirror stage.

Moreover, the cutting of the cross-cap detaches a right-handed or left-handed Moebius strip, depending on how it is unfolded, and a single object (a), the disk--more precisely an immersion of the disk, a disk provided with a line of intersection, as is shown in these drawings:



The numbers on the flattened surfaces of the disk refer to the order of the mounting of the triangles in the eight-cone.

We must then interiorize the loop of the eight and look at the cone from above, having given our gaze a quarter-twist:



The right-left disparity has no role in these occurrences.

On this disk is situated the point Φ insuring the cross-cap structure.

It is appropriate to evoke here the three modalities of the drive established in grammatical terms by Freud: the passive, the active, and the pronominal (the middle voice of ancient Greek). Not that these differences are recovered, but rather that language translates according to its means a disparity situated elsewhere topologically. Because the unfolding of these three objects, after the cutting of the fantasy--their being plunged into our ordinary space--produces a left-handed Moebius strip, a right-handed Moebius strip, and a bilateral disk.

The imaginary couple of the mirror stage leaves in the hollow (*en creux*) the place marked by the point Φ , which allows us to subsume within this organization called the cross-cap all of the objects (a).

Beyond the imaginary identification with his image, the subject is also identified with a third term, "that of the ternary imaginary, where the subject identifies with the opposite of his living being, which is nothing other than the phallic image whose unveiling in this function is not the least scandalous aspect of the Freudian discovery" (*Traitement*).

According to Freud, the imaginary function presides over the investment of the narcissistic object. We have shown that "the specular image is the channel taken by the transfusion of the libido of the body toward the object."

"But insofar as a part (of the libido) remains preserved from this immersion, concentrating in itself the most intimate aspect of autoeroticism. Its 'plunging' (*en pointe*)⁵ position in the form predisposes it to the fantasy of an outmodedness where is achieved the exclusion where it is found the specular image and the prototype it constitutes, for the world of objects. It is in this way that the erectile organ comes to symbolize the place of jouissance, not as itself, nor even as an image, but as a part lacking in the desired image."⁶

This quotation reminds us that at the moment itself when the structures of the object (a) are diversified, the point Φ remains central to the functional organization of this object, which differentiates the drives.

On the sphere and the torus, the phallus must be sought in the Other, present with all the carnal weight of a body in the organization of the drive. On the cross-cap, it is in the right place (*de droit*), we will say.

Finally, it must be positioned on the Klein bottle.

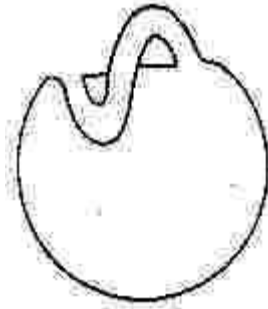
The object (a) remains a point of articulation between fantasy, drive, and the "passions of being" (love, hatred, or ignorance). Excluded from the specular image, it nonetheless constitutes a hole in the organization of the Ego; it permits a knotting with something of the Other, from the exterior.

We have already evoked the sphere, the torus, and the cross-cap. It is *apropos* of the problem of the mirror and of the twist that we introduce the Klein bottle, a topological object associated by Lacan with the voice.

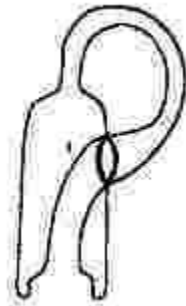
It is a question of a sphere on which a tunnel becomes a handle: (see photo)

⁵ In *haute couture*, "*en pointe*" refers to a plunging neckline (*décolleté*) (Tr.).

⁶ *Subversion du sujet et dialectic du desir*.



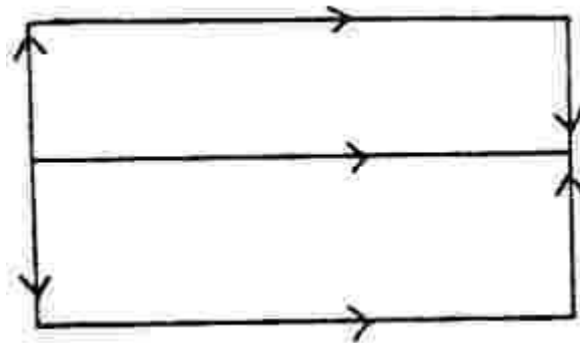
We can also describe it as a bottle whose base joins its neck, producing an intersection in the form of a circle:



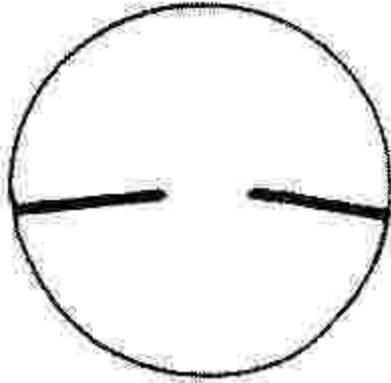
This structure makes appear a space whose interior is continuous with the exterior. Only a trajectory differentiates them. Since it retains a central hole, it is associated with the torus. Topologists show how a torus can be rolled as a double surface around a Klein bottle.

They also say that it constructed from two Moebius strips joined along their edge, but two strips with different twists:

(The torus is also reconstituted from two Moebius strips, but with an identical twist.)



The Klein bottle can in theory be represented by a sphere to which are added two intersection like that of the cross-cap.



Is this based on the fact that we can position the point Φ on the Klein bottle, in a doubling of castration, that of the subject and the Other?

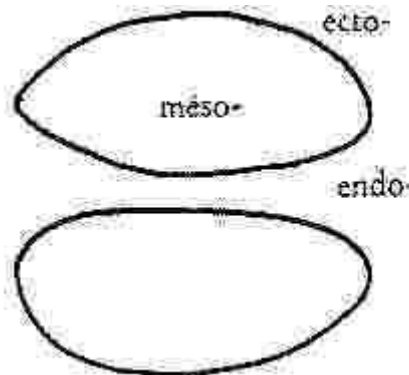
Lacan considers, in fact, that "nowhere is the subject more interested in the Other than by this object." Clinically, moreover, remember that the voice is the object Lacan puts at the center "of the relations between the sadist and the masochist." He develops it thusly: "Masoch organizes things so as to have no more speech; he signs some contracts enjoining him to have nothing more to say. The sadist tries to de-complete the other by withdrawing speech from him and imposing his voice on him."⁷

By way of these objects (a), the body is present by its orifices. The holes of the organism offer their edge to structures accounted for by topological structures. They are organizations of the hole, and they give form to the space of the hole.

Hence, the voice takes up two bodily orifices in the same structure, the ear to hear and the mouth to speak.

The gaze puts in place a structure so particular that we have to make use of the joke to account for it: it is a wonder "that cats have two holes cut in their skin precisely where their eyes are." (We recognize here one of Freud's examples in his study on jokes).⁸ The torus encloses the mouth and the anus in the same organization.

These are the two orifices of the same hole: the gut or the digestive tract. Also at issue is the physical structure of the set (*l'ensemble*) of the body: ectoderm, mesoderm, endo . . .



⁷ Seminar of March 26, 1969, unpublished, *D'un Autre à l'autre*.

⁸ Sigmund Freud, Jokes and Their Relation to the Unconscious, translated by James Strachey (New York: W. W. Norton & Co., 1963), p. 59 (Tr.).

The sphere constructs the hole on its denial. The breast completes the mouth of man's child. The psychic organization of the subject then refers to a totality in which the breast detached from the mother forms a part.

The coupling two-by-two of objects (a) finds another *raison d'être*. The cross-cap and the Klein bottle are structures of unilateral space. They put interior and exterior in continuity. The voice and the gaze are on the body, by the quite specific ways of sensorial organs. At issue are the only passages of exterior space into mesodermic space. Only a unilateral, Moebien organization of space can account for this.⁹

The four objects (a) are without specular image, because they are holes, henceforth specifically organized.

There are thus two kinds of objects dividing human knowledge: those to which the mirror gives an identity and a substantiality that is only the reflection of the Ego, and those that plug up the hole, organizing it, veiling it. The failure (*l'échec*) of this function makes castration anxiety well-up or spring forth.

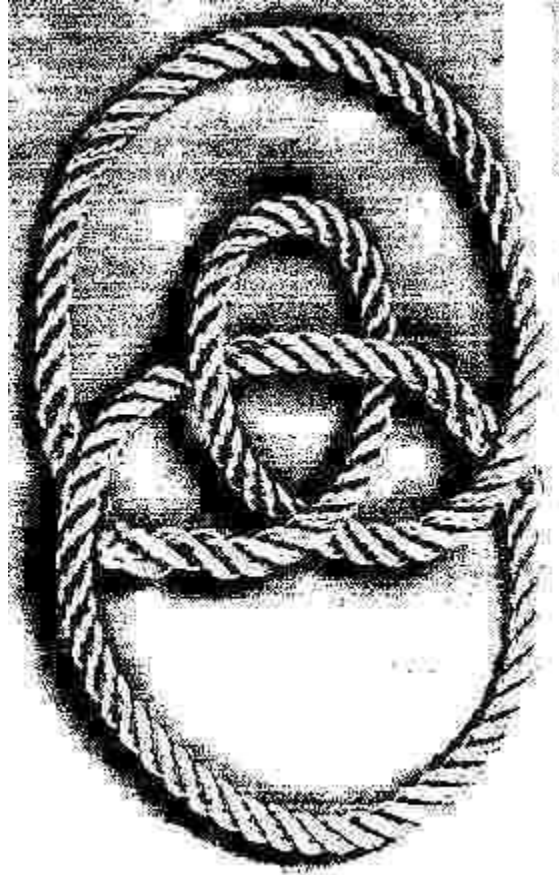
Clinically, Lacan came to distinguish between two imaginaries: "the true and the false."¹⁰

The false imaginary pertains to the necessary illusions of the mirror, the true to the fantasy, to desire, to anxiety.

This is why Lacan always opposed *i(a)*, the image of the other, and the object (a).

⁹ Seminar of May 16, 1962, unpublished, *L'Identification*.

¹⁰ Seminar of June 13, 1962, unpublished, *L'Identification*.



The so-called knot of the fantasy.