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## Base Definitions

### *Characteristic*

is defined beginning with Euler's theorem as vertexes (*sommets*) - edges (*aretes*) + faces = X. This X is the characteristic of a surface.

Obviously, to be able to calculate it, we must give to the surfaces a form provided with vertexes, edges, and faces. This presentation is called "rigidified in planes" ("*en rigides par plaques*").

### *Classification of surfaces*

There exist several fashions to classify topological surfaces, according to their genre or characteristic.

There are two large families of surfaces: non-orientable surfaces and orientable surfaces. Two different surfaces can have the same characteristic.

Here is the table for non-orientable surfaces:

Characteristic:

|                       |  |                    |                    |
|-----------------------|--|--------------------|--------------------|
| 1<br>projective plane | 0<br>Klein bottle<br>or 2-plane projective | -1<br>3-projective | -2<br>4-projective |
|-----------------------|--|--------------------|--------------------|

and for orientable surfaces:

|                              |            |                                     |               |
|------------------------------|------------|-------------------------------------|---------------|
| 2<br>sphere or zero<br>torus | 0<br>torus | -2<br>two-holed torus<br>or 2-torus | -4<br>3-torus |
|------------------------------|------------|-------------------------------------|---------------|

Between these two families, there is a sheath relation (*relation de doublure*), which means that orientable surface can envelop the corresponding non-orientable surface.

### *Dual cuts or section couples*

These are two cuts made on a surface, which have only one point in common. Thus, on the torus there is a section couple.

### *Dimensions*

Each of the sizes necessary for the evaluation of figures and solids. We often define time with the term fourth dimension.

### *Genre*

The maximum number of closed, disjointed lines that can be made on a surface without fragmenting it (or cut).

This number permits a classification of surfaces.

### *Immersion and plunging (plongement)*

Our space is three-dimensional. One can speak of an immersion of a surface as soon as one makes an abstraction of this space and makes such impossible phenomena as resectioning (*recoupe*) or the triple point intervene in our space . . .

On the other hand, a surface is plunged when it does not make an abstraction of its space, of the environment (*milieu*) where it is. A sheet of paper constitutes an environment, just as does our everyday three-dimensional space.

### *Intrinsic and Extrinsic*

A property is intrinsic to a surface when it is maintained whatever its space of plunging.

A property is extrinsic when it depends on the plunging space of a surface.

For mathematicians, the twist is an extrinsic property. (All that matters is knowing whether the number is even or odd.)

### *Moebian, Moebian space*

A hasty denomination of the space proper to the projective plane. Its immersion known by the term "cross-cap," constructed on a Moebius strip with one half-twist, allows us to understand this usage.

Thus, this adjective is often used as a synonym for non-orientable or unilateral.

### *Orientable and Non-orientable*

As soon as we leave surfaces in two dimensions, the concept of unilateral no longer functions.

Orientation then comes into play.

To define it, we must return to the law discovered by Moebius (and which specifically permitted him to discover non-orientable surfaces).

We begin with a tetrahedron (a polyhedron composed of four triangles).

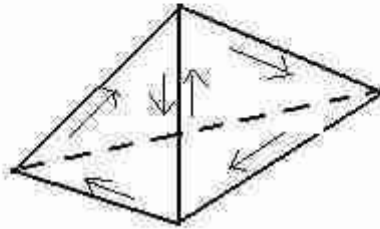
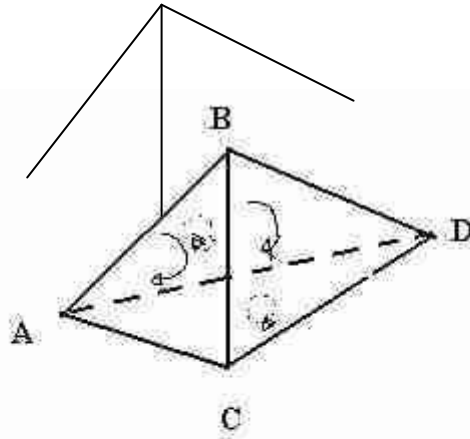
We define a direction of reading the vertexes of the triangles composing the polyhedron.

When we turn the orientation of our reading in the same direction for all of the polyhedron's triangles, the edges are crossed in an opposed direction depending on whether consider them part of one face or the adjacent face.

This quality is an invariant of a surface.

(We should add that all polyhedrons are decomposable into triangles.)

Let us also note that this direction is not the same for the observer placed at the exterior of the polyhedron.



On the other hand, a surface is called non-orientable when, in this decomposition into triangles of a polyhedron, two edges are found to be not oriented in the same direction (cf. the heptahedron of Reinhart).

*Enveloment of two thicknesses (Revetement á deux feuillets)*

A topological manipulation that consists in giving to a surface the form of an envelopment of two thicknesss from another surface. When the surface is in this position, on can, without violating the law of continuous transformation, make these two thicknesses stick together and transform the first surface, doubled, into the second.

This procedure serves when one can produce a turning-inside-out of an orientable surface. This envelopment of two thicknesses is a symetrical point of the process.

*Continuous transformation*

This is the operation founding the equality of surfaces in topology. Two surfaces are said to be identical when one can transform one into the other by continuous transformations in the domain of plungings.

It is defined by the existence, always possible, of a tangent on a curve that varies in a continuous fashion.

*Unilateral or Bilateral*

Said of a surface depending on whether it has a single edge or two edges. This is the concept Moebius brought to light in discovering the strip that bears his name.

This ribbon of Moebius is unilateral; one can also say that it is non-orientable.

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